(A semi comprehensive) Network analysis of an exchange program

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(A semi comprehensive) Network analysis of an exchange program

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POINTS:

- Positional analysis
- Graph-level indices
- Network evolution

PROSPECTS - ***when network order differs..:***

- Assessing change in positional systems
- Indices trajectories over time
- (Specification of the SIENA model)

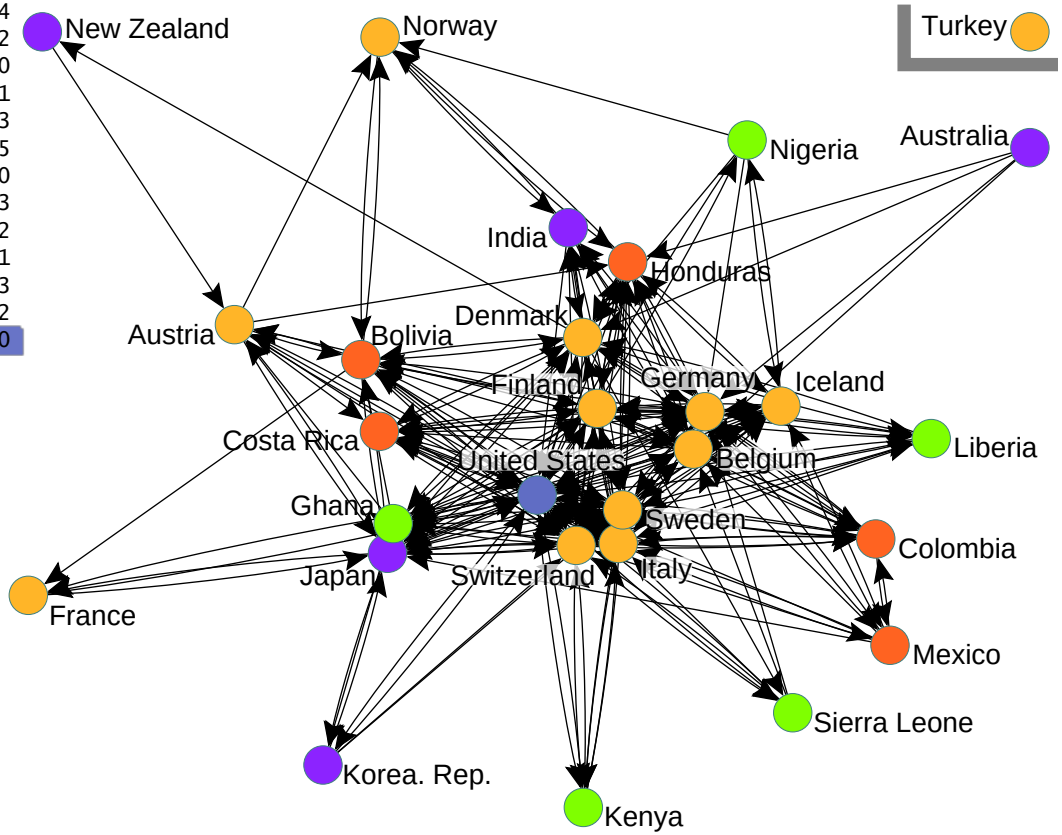
- Conclusions

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Country \ Year	75-76	76-77	77-78	78-79	79-80	80-81	81-82	82-83	83-84	84-85	85-86	86-87	87-88	88-89	89-90	90-91	91-92	92-93	93-94	94-95	95-96	96-97	97-98	98-99	99-00	00-01	01-02	02-03	03-04	04-05	05-06
EUROPE (EU)																															
Au Austria	•	NA			•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Be Belgium	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Dk Denmark	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Fi Finland	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Fr France	•	NA				•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
Ge Germany*	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Is Iceland	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
It Italy		NA		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Lt Lithuania		NA																										•	•	•	•
Nl Netherlands	•	NA																		•	•	•	•								
Ni Northern Ireland	•	NA																													
No Norway		NA								•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•		•	•	•	•	
Po Poland		NA																•	•	•	•	•	•								
Pt Portugal		NA																											•		•
Ru Russia		NA																										•	•	•	•
Sp Spain		NA																•	•	•	•	•	•	•	•	•	•				
Se Sweden	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•				•	•	•	•	•	•	•	•	•	•
Sw Switzerland	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Tu Turkey		NA										•	•																		
Uk United Kingdom		NA																		•	•	•	•	•	•	•	•	•	•	•	•
LATIN AMERICA (LA)																															
Bo Bolivia	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Br Brazil	•	NA												•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Co Colombia		NA						•	•	•	•	•		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Cr Costa Rica	•	NA			•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Gu Guatemala	•	NA																													
Hn Honduras		NA						•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Mx Mexico	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Nc Nicaragua		NA							•	•	•																				
Ur Uruguay		NA																					•								
ASIA-PACIFIC (AP)																															
At Australia	•	NA	•	•	•	•	•	•	•	•	•	•	•		•	•	•	•	•	•											
Hk Hong Kong	•	NA																													
In India	•	NA						•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Id Indonesia	•	NA	•	•	•	•																									
Jp Japan	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Kr Korea, Rep. of	•	NA											•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Np Nepal		NA																										•	•	•	•
Nz New Zealand	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Ph Philippines	•	NA	•	•	•	•	•																								
Tw Taiwan		NA																•	•	•	•	•	•	•	•	•	•	•	•	•	•
Th Thailand		NA																										•	•	•	•
AFRICA (AF)																															
Gh Ghana		NA				•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Ke Kenya		NA											•	•	•	•	•			•	•	•	•	•	•	•	•	•	•	•	•
Li Liberia		NA								•	•	•	•	•	•	•	•														
Ma Morocco		NA																										•	•	•	•
Mz Mozambique		NA																										•	•	•	•
Ng Nigeria		NA							•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Sl Sierra Leone		NA											•	•	•	•	•	•	•	•	•	•	•								
Sa South Africa	•	NA																													
Ug Uganda		NA																										•	•	•	•
NORTH AMERICA (NA)																															
Ca Canada		NA								•	•							•	•	•	•							•	•		
Us United States	•	NA	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•								

* West Germany until year 1990. NA = data not available.

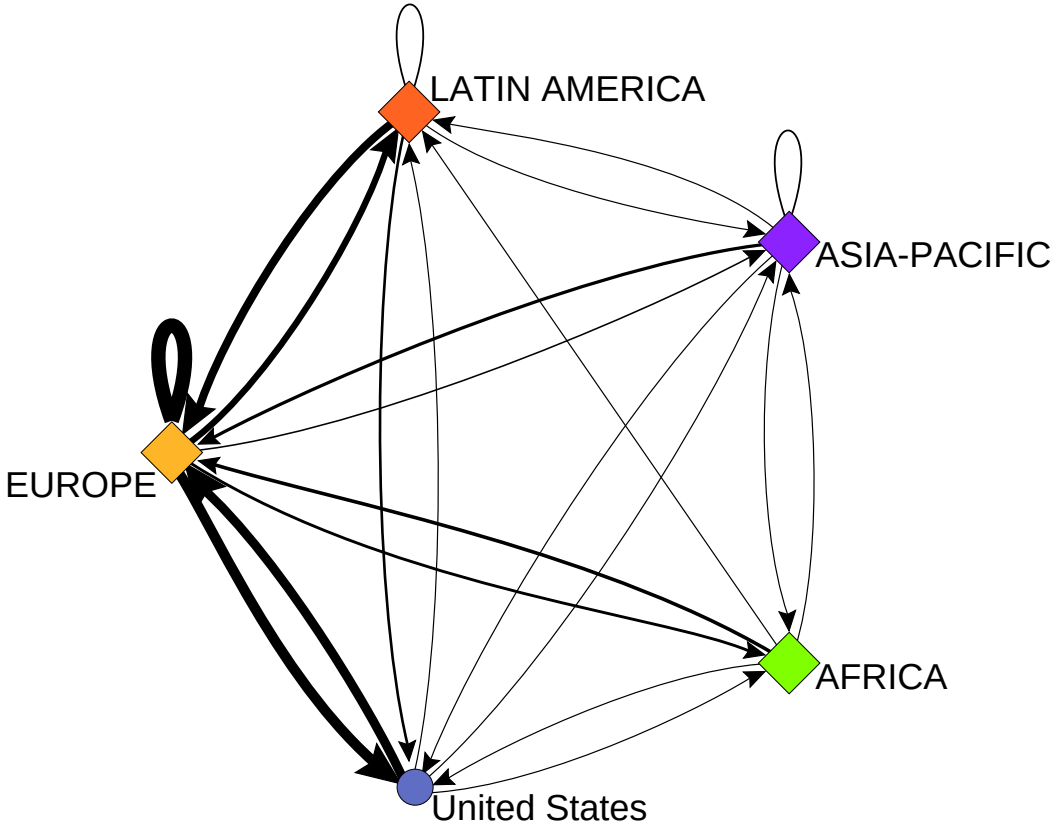
The Long-term program network - t_{12} (year 1987-88)

	Au	Be	Dk	Fi	Fr	Ge	Is	It	No	Se	Sw	Tu	Bo	Co	Cr	Hn	Mx	At	In	Jp	Kr	Nz	Gh	Ke	Li	Ng	Sl	Us
Au	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	4
Be	0	0	0	1	0	0	2	0	0	0	0	0	1	1	1	1	0	0	1	1	0	0	0	0	0	0	0	1
Dk	0	0	0	0	0	1	0	2	0	0	1	0	1	2	1	2	0	0	2	1	0	1	1	0	1	0	1	5
Fi	1	2	0	0	0	6	1	3	0	0	2	0	0	1	1	3	0	0	0	2	0	0	1	0	1	0	0	14
Fr	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	3
Ge	0	0	1	5	0	0	2	9	0	2	1	0	3	4	2	3	2	0	2	2	0	0	1	0	2	0	0	29
Is	0	1	0	1	0	2	0	1	0	1	1	0	0	0	0	1	1	0	0	0	0	0	2	0	0	1	0	1
It	1	0	2	3	0	10	1	0	0	2	1	0	0	1	1	0	1	0	0	2	0	0	0	1	0	0	0	14
No	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
Se	0	1	0	0	0	3	1	2	0	0	2	0	1	2	1	3	0	0	1	1	0	0	1	1	1	0	0	5
Sw	0	0	0	1	0	0	1	0	0	1	0	0	2	3	1	1	0	0	1	1	0	0	4	1	1	0	3	10
Tu	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Bo	1	1	1	1	1	3	0	1	1	1	3	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	4
Co	0	1	1	1	0	4	0	2	0	2	3	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	5
Cr	0	0	1	1	0	3	1	2	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7
Hn	0	1	2	3	0	4	1	2	1	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
Mx	0	1	0	0	0	4	2	1	0	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	2
At	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
In	0	1	1	0	0	3	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Jp	1	1	2	2	1	3	0	1	0	1	1	0	1	0	0	0	0	0	0	0	1	0	1	0	0	0	0	3
Kr	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	5
Nz	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Gh	1	0	1	2	0	1	2	1	0	1	2	0	2	0	0	0	0	0	0	1	1	0	0	0	0	0	0	3
Ke	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
Li	0	0	1	1	0	3	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Ng	0	0	1	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
Sl	0	2	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
Us	4	1	5	13	3	19	1	14	0	6	10	0	1	2	5	2	0	0	0	3	3	0	1	2	1	1	2	0



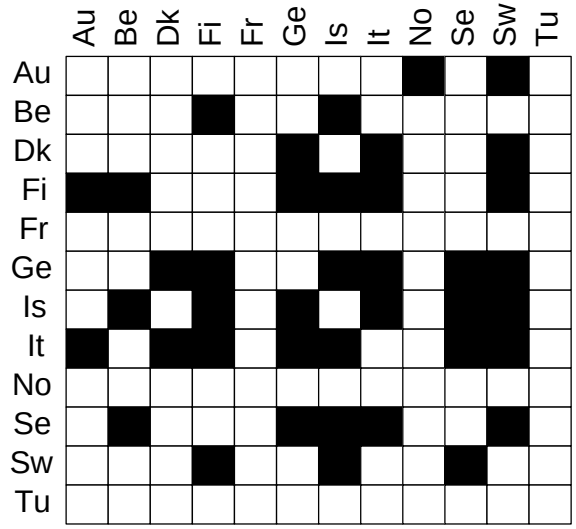
*Turkey has been moved closer to get it into the picture.

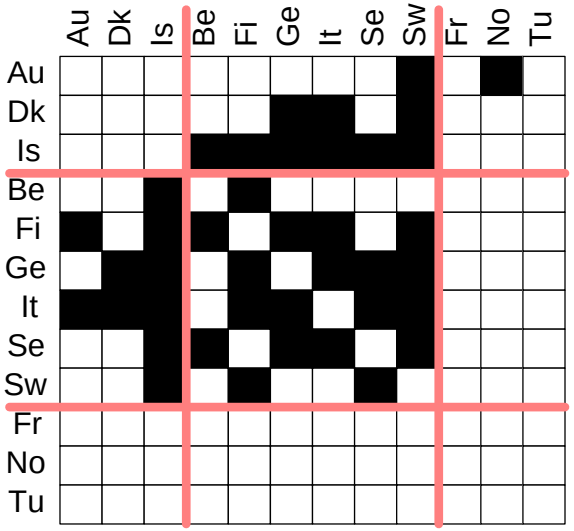
Network reduction by its composition



	EU	LA	AP	AF	Us
EU	83	53	13	24	86
LA	67	2	2	0	22
AP	27	2	2	1	9
AF	30	2	2	0	11
Us	76	10	6	7	0

t_{12}





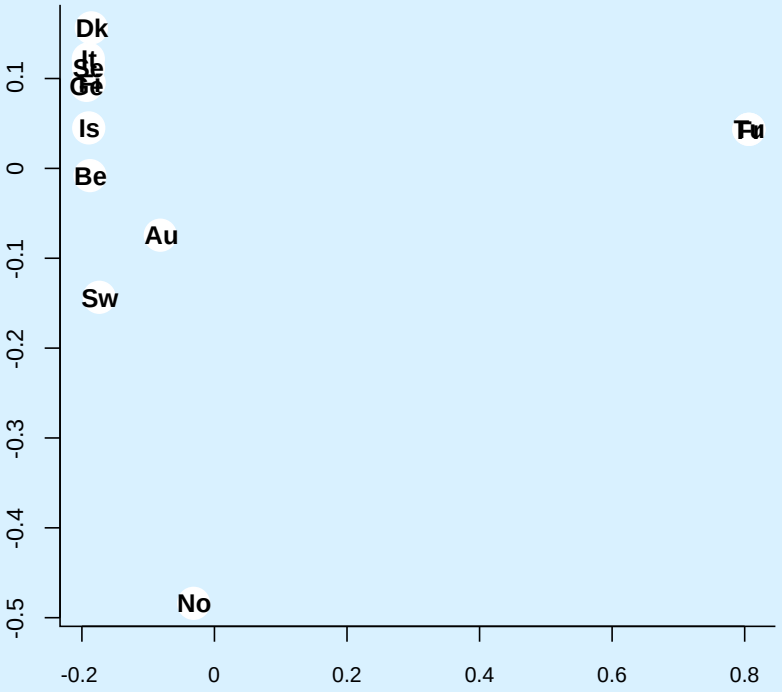
null reg null
reg reg null
null null null

↓

	1	2	3
1	0	.556	.037
2	.556	.528	0
3	0	0	0

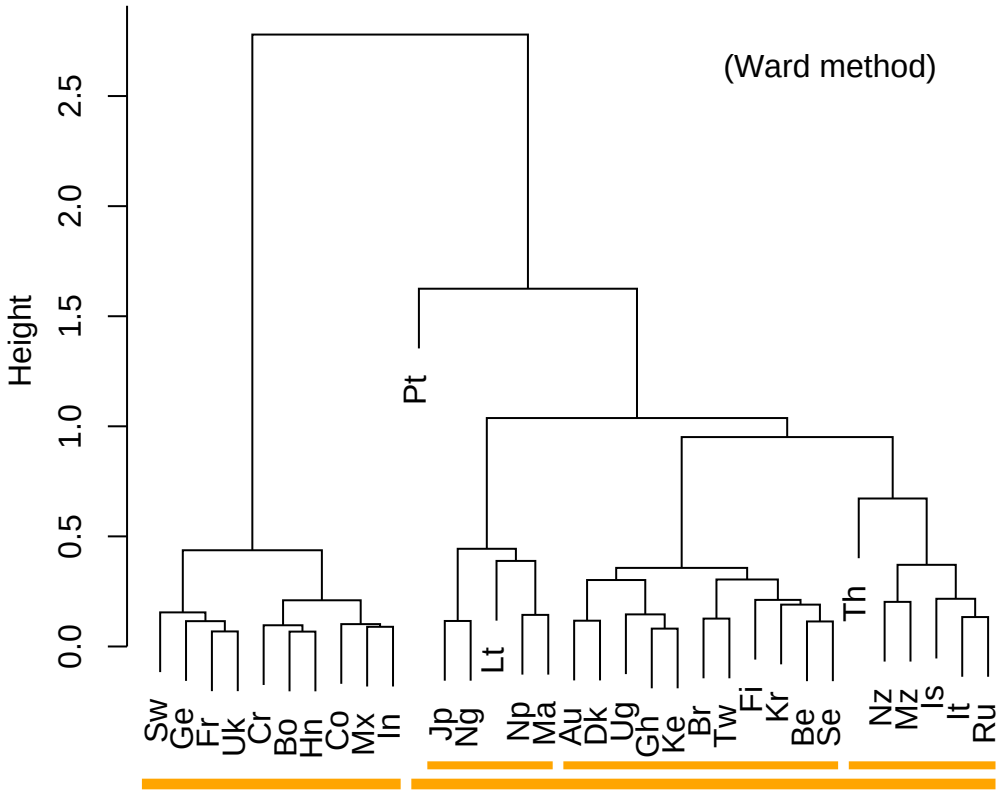
↓

null com null
com com null
null null null

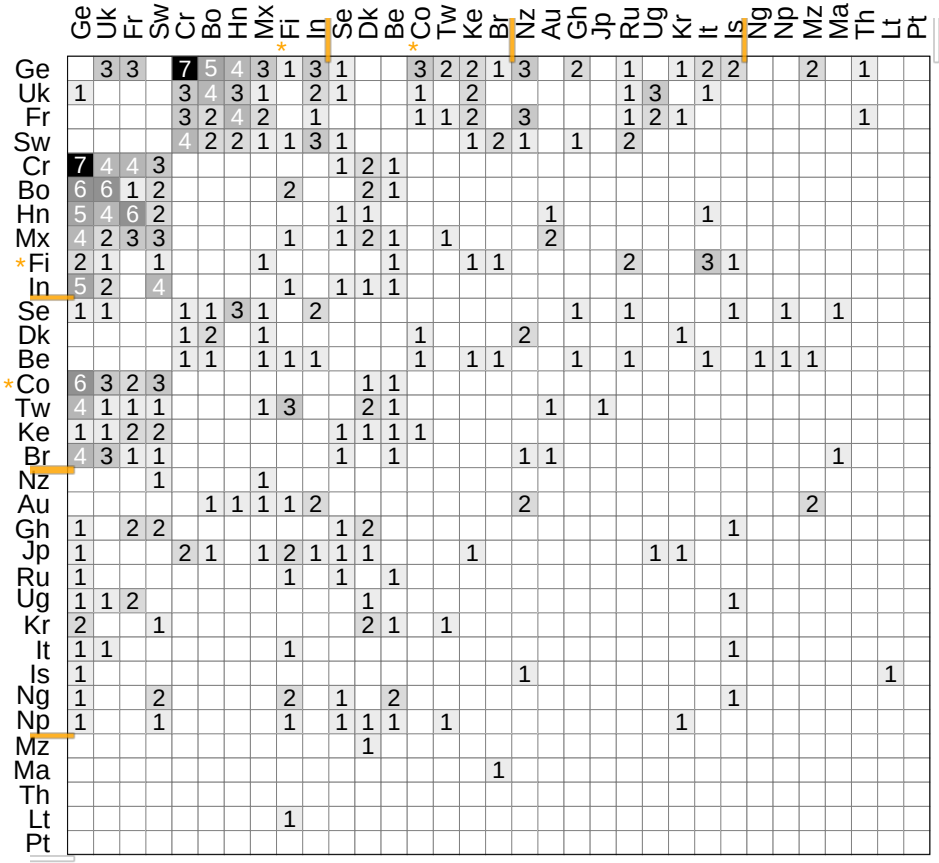


$$M_{ij}^{t+1} = \frac{\sum_{k=1}^{n} \max_{m=1}^{n} \sum_{r=1}^{R} M_{km}^{t} \left({}_{ijr} M_{kmr}^{t} + {}_{jir} M_{kmr}^{t} \right)}{\sum_{k=1}^{n} \max_{m}^{*} \sum_{r=1}^{R} \left({}_{ijr} \mathrm{Max}_{kmr} + {}_{jir} \mathrm{Max}_{kmr} \right)} \qquad \text{(REGE)}$$

	Au	Be	Dk	Fi	Fr	Ge	Is	It	No	Se	Sw	Tu
Au												
Be	.46											
Dk	.49	.24										
Fi	.56	.18	.10									
Fr	1	1	1	1								
Ge	.49	.15	.02	.07	1							
Is	.63	.12	.11	.05	1	.07						
It	.49	.20	.04	.05	1	.02	.11					
No	.70	.55	.67	.60	1	.61	.54	.64				
Se	.51	.19	.05	.05	1	.03	.06	.05	.63			
Sw	.40	.05	.36	.28	1	.25	.22	.28	.43	.30		
Tu	1	1	1	1	0	1	1	1	1	1	1	

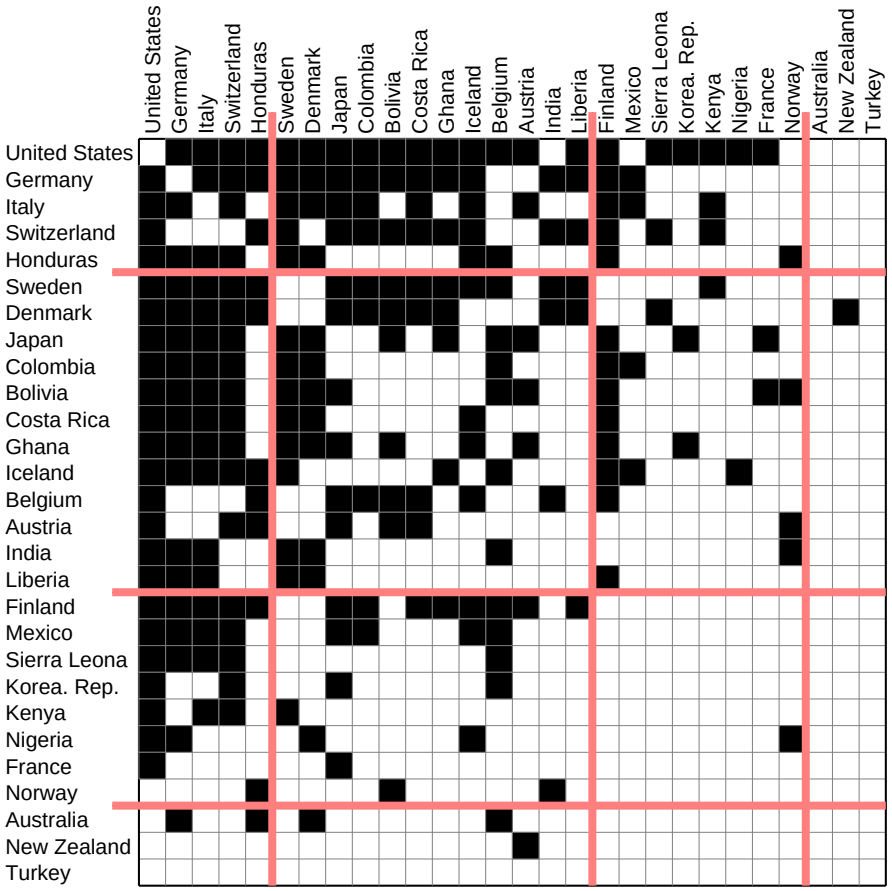


Hierarchical clustering



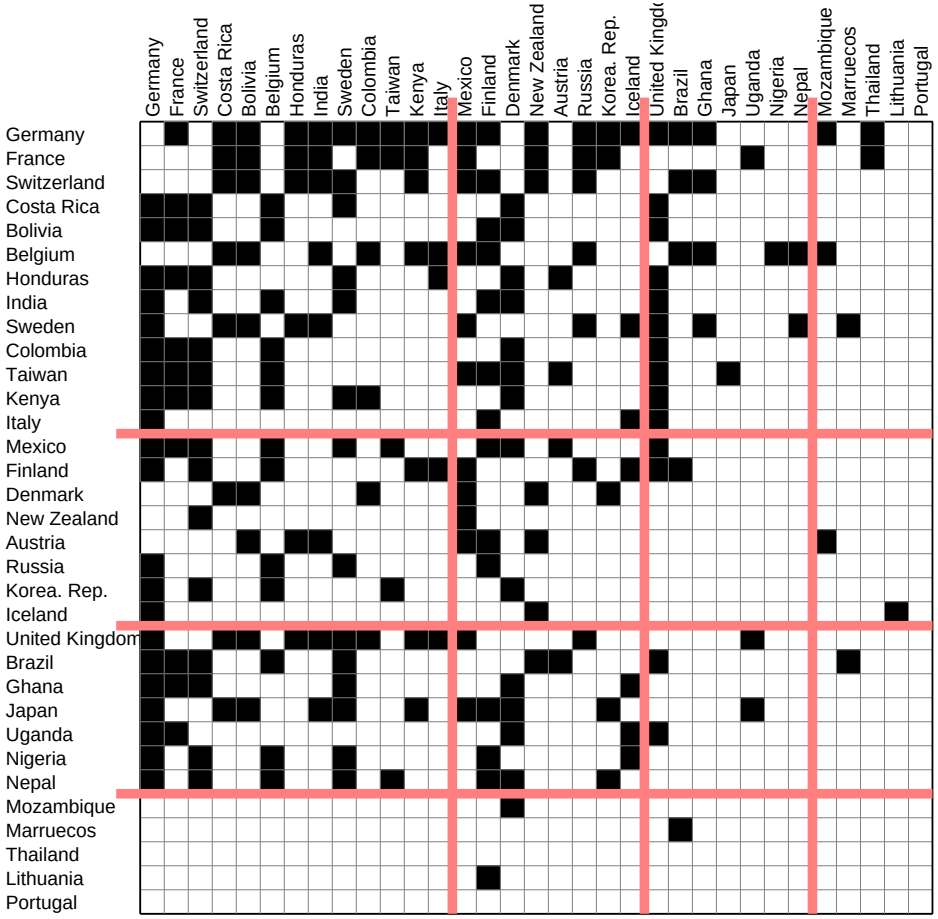
Ranked matrix

t_{30}



t_{12}

com	com	reg	null
com	reg	reg	null
com	reg	null	null
null	null	null	null



t_{30}

reg	reg	reg	null
reg	reg	null	null
reg	reg	null	null
null	null	null	null

Period of time:	t_{12}	t_{30}
Positional systems proposed		
Blocks: complete	4	0
regular	4	7
null	8	9
Error measurement	7	18
Normalized error	.028	.076
Number of actors	28	33
Number of ties	246	236
Density, dichotomous relation	.325	.223

	Au	Be	Dk	Fi	Fr	Ge	Is	It	Lt	No	Pt	Ru	Se	Sw	Tu	Uk	Bo	Br	Co	Cr	Hn	Mx	At	In	Jp	Kr	Np	Nz	Tw	Th	Gh	Ke	Li	Ng	Sl	Ug	Us
t_{12}	B	B	B	C	C	A	B	A	X	C	X	X	B	A	D	X	B	X	B	B	A	C	D	B	B	C	X	D	X	X	B	C	B	C	C	X	A
t_{30}	B	A	B	B	A	A	B	A	D	X	D	B	A	A	X	C	A	C	A	A	A	B	X	A	C	B	C	B	A	D	C	A	X	C	X	C	X

		t_{30}				
		A	B	C	D	X
t_{12}	A	4	0	0	0	1
	B	6	3	2	0	1
	C	2	3	1	0	2
	D	0	1	0	0	2
	X	1	1	4	3	0

The Rand index (Rand, 1971)

$$R_{(P,Q)} = \frac{\binom{n}{2} + \sum_{i=1}^{R}\sum_{j=1}^{C} x_{ij}^2 - \frac{1}{2}\left[\sum_{i=1}^{R} x_{i\bullet}^2 + \sum_{j=1}^{C} x_{\bullet j}^2\right]}{\binom{n}{2}} \quad ; \quad \delta = 1 - R_{(P,Q)}$$

		t_{12}				
		A	B	C	D	X
G	EU	3	5	3	1	4
	LA	1	3	1	0	1
	AP	0	2	1	2	3
	AF	0	2	3	0	1
	Us	1	0	0	0	0

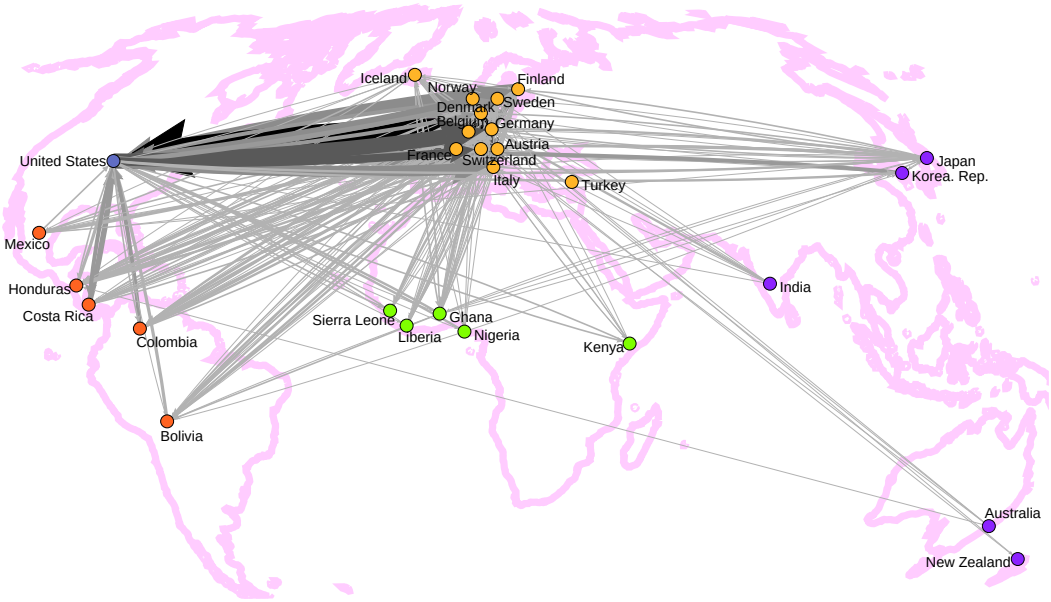
		t_{30}				
		A	B	C	D	X
G	EU	6	5	1	2	2
	LA	4	1	1	0	0
	AP	2	2	2	1	1
	AF	1	0	3	0	2
	Us	0	0	0	0	1

Partition	R	δ
t_0 - t_{12}	.638	.362
t_0 - t_{30}	.613	.387
t_{12} - t_{30}	.672	.328
t_0 - G	.609	.391
t_{12} - G	.594	.406
t_{30} - G	.614	.386

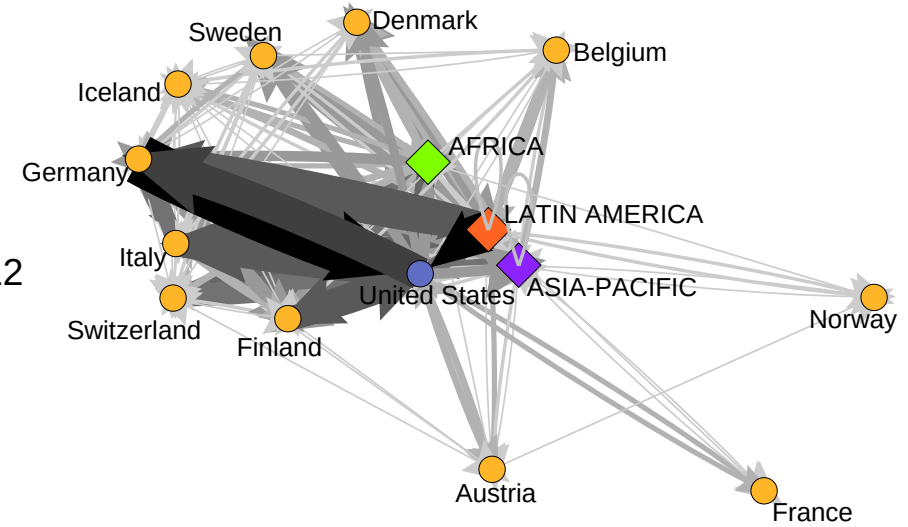
$n = 51$

Partition	R	δ
t_{12} - t_{30}	.688	.312
t_{12} - G	.620	.380
t_{30} - G	.635	.365

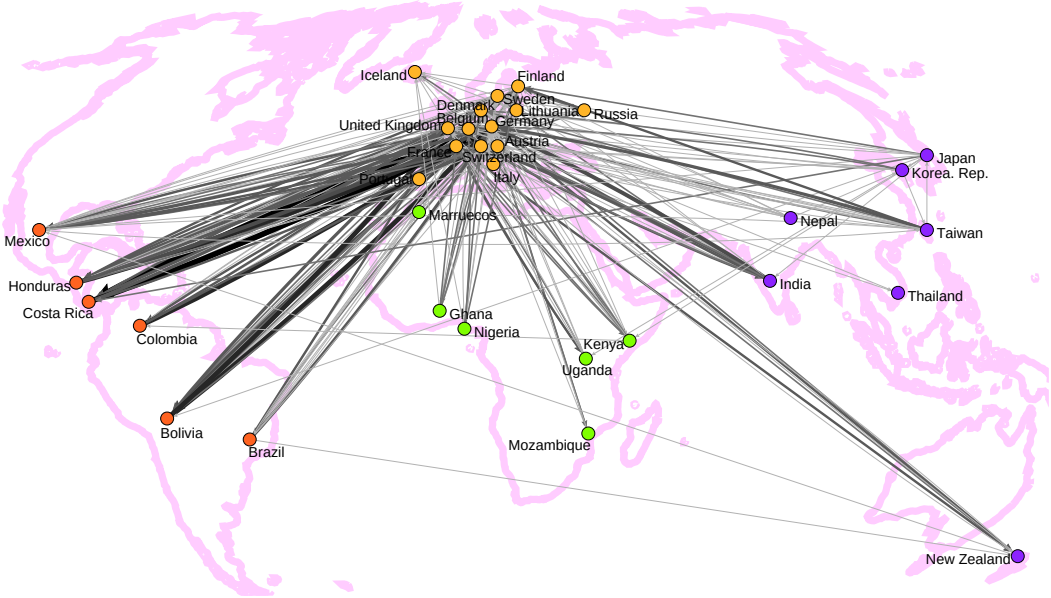
$n = 37$



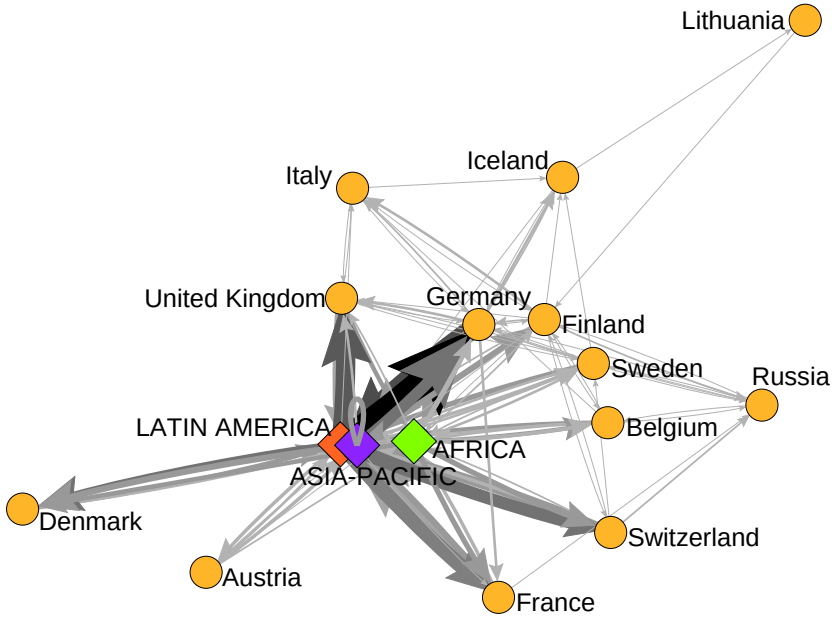
t_{12}



*Turkey is disregarded here due is isolated.



t_{30}



*Portugal is disregarded here due is isolated.

- A graph $\mathcal{G}$ is defined as a set of vertices $\mathcal{V}$ and a set of segment lines $\mathcal{L}$ between vertices; $\mathcal{G}(\mathcal{V},\mathcal{L})$ for $\mathcal{V} = \{v_1, v_2, ..., v_n\}$, and $\mathcal{L} = \{l_1, l_2, ..., l_L\}$, where n is the number of actors in $\mathcal{V}$, and L is the number of lines in $\mathcal{L}$.
- x_{ij} records the presence or absence of a tie from vertex v_i to vertex v_j.

$$\Delta = \frac{L}{n(n-1)}$$

Density

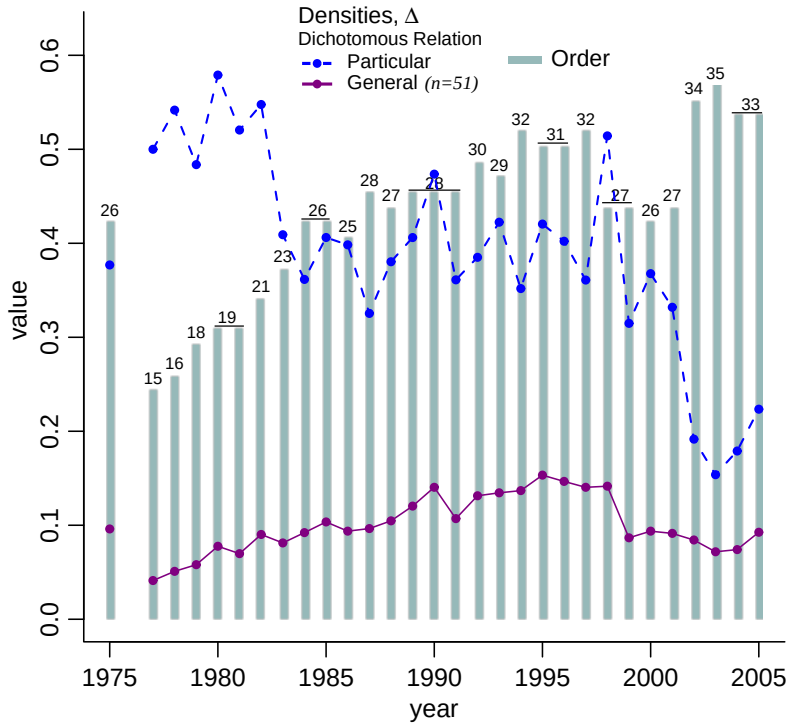
$$C_D = \frac{\sum_{i=1}^{n} c_D(v^*) - c_D(v_i)}{[(n-1)(n-2)]}$$

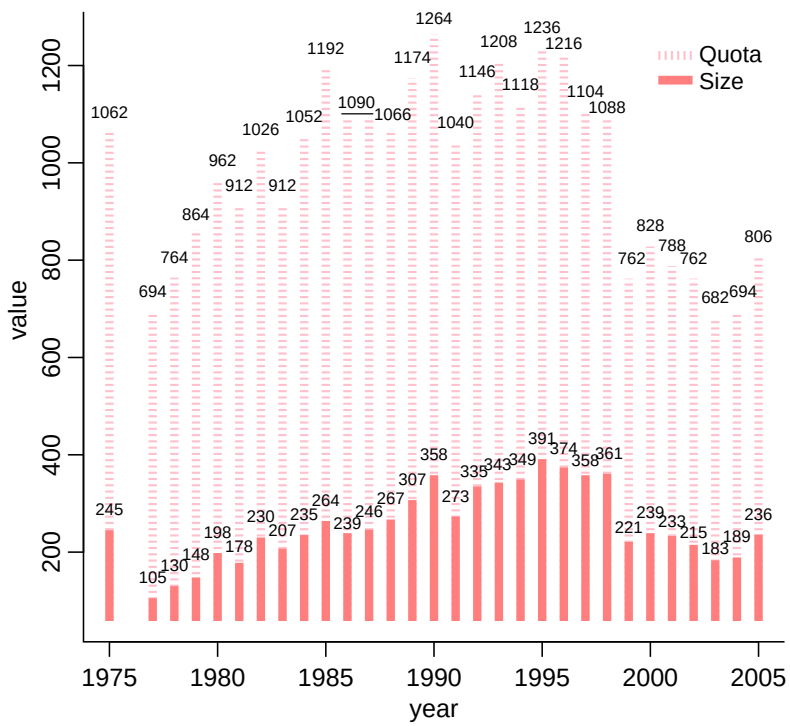
Centralization

1. $c_D(v_i) = \frac{x_{\bullet i} + x_{i\bullet}}{2(n-1)}$
2. $p_D(v_j) = \frac{x_{\bullet j}}{n-1}$
3. $c_D^{OUT}(v_i) = \frac{x_{i\bullet}}{n-1}$

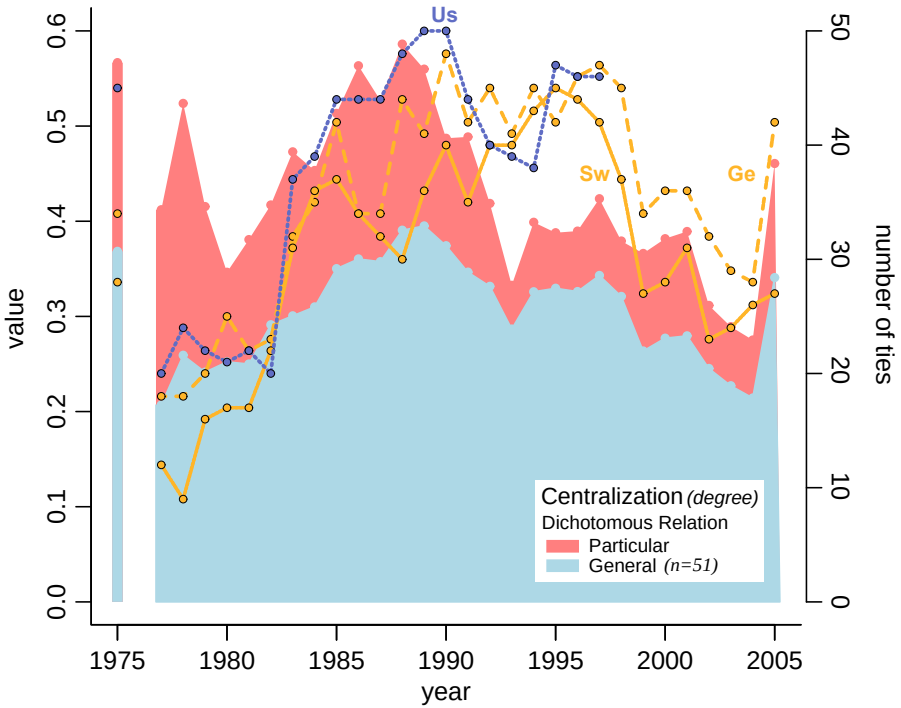
Centralities

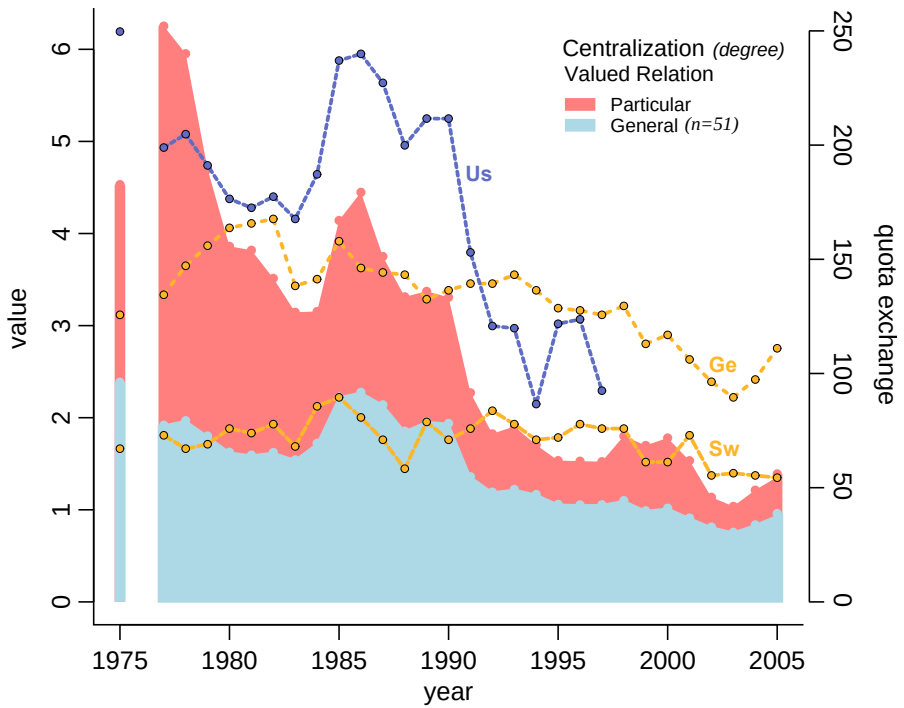
		total	avg.	s.d.	min.	max.
Times recorded (m):		30				
Actors involved (n):		51	26.7			
Exchange relations:		14258	475.3			
Quota of exchange:		29602	986.7			
DENSITY	dichotomous, Δ:	.390	.107	.154	.579	
	valued:	.815	.362	.287	1.652	
CENTRALIZATION	dichotomous:	.443	.090	.275	.619	
	valued:	2.838	1.449	1.037	6.253	





Graph-Level Indices Trajectories



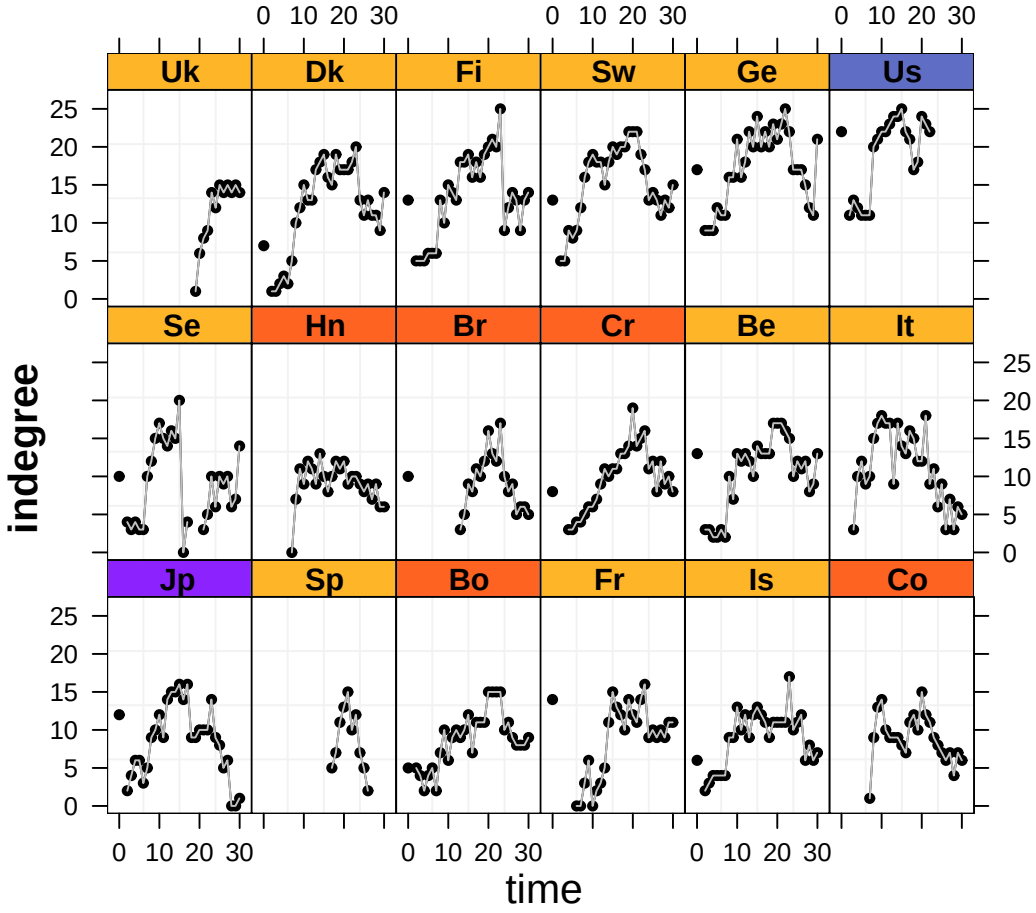


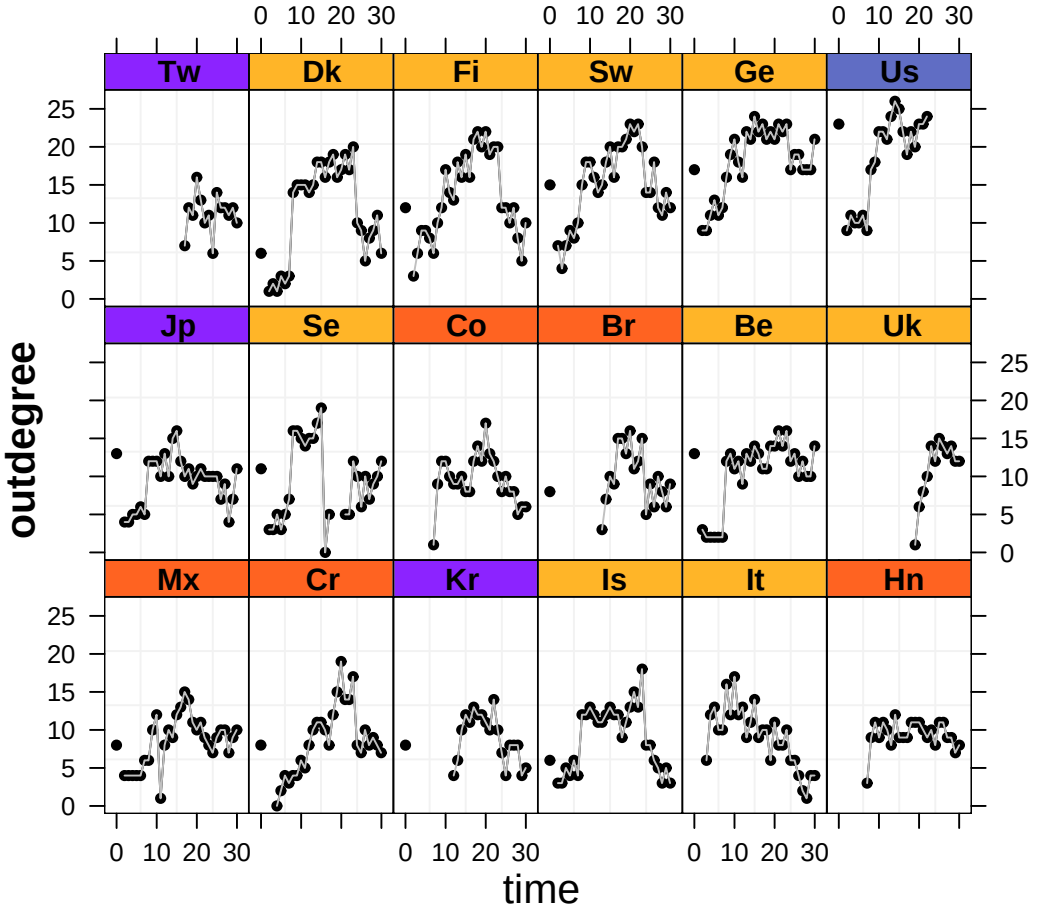
Network Transitions

obs. times	0 → 0	0 → 1	1 → 0	1 → 1	Total	Distance	Missing	$\hat{\lambda}$	(s.e.)
0 to 2	73	33	173	72	351	206	.92	19.71	(2.72)
2 to 3	73	47	22	83	225	69	.92	10.30	(2.07)
3 to 4	81	44	26	104	255	70	.91	9.15	(1.75)
4 to 5	112	54	4	144	314	58	.88	5.01	(0.84)
5 to 6	85	30	50	148	313	80	.88	8.01	(1.28)
6 to 7	90	70	18	160	338	88	.88	8.87	(1.39)
7 to 8	166	35	58	172	431	93	.84	8.68	(1.25)
8 to 9	240	69	41	166	516	110	.80	8.57	(1.12)
9 to 10	343	72	43	192	650	115	.75	7.34	(0.95)
10 to 11	249	45	70	194	558	115	.78	8.68	(1.15)
11 to 12	310	74	67	172	623	141	.76	11.52	(1.48)
12 to 13	332	82	61	185	660	143	.75	9.54	(1.14)
13 to 14	351	90	50	217	708	140	.72	9.77	(1.11)
14 to 15	351	98	47	260	756	145	.70	9.61	(1.13)
15 to 16	348	50	135	223	756	185	.70	15.00	(1.69)
16 to 17	307	132	70	203	712	202	.75	16.46	(1.94)
17 to 18	401	85	77	258	821	162	.68	9.00	(0.90)
18 to 19	397	92	86	257	832	178	.68	11.24	(1.20)
19 to 20	479	102	60	289	930	162	.64	9.25	(0.96)
20 to 21	425	80	97	294	896	177	.66	10.72	(1.10)
21 to 22	484	79	95	279	937	174	.64	9.40	(0.94)
22 to 23	305	99	96	262	762	195	.72	12.17	(1.23)
23 to 24	327	14	154	207	702	168	.72	12.67	(1.37)
24 to 25	360	69	51	170	650	120	.75	7.48	(0.91)
25 to 26	358	54	60	179	651	114	.75	6.62	(0.76)
26 to 27	367	52	70	163	652	122	.75	7.26	(0.84)
27 to 28	864	43	75	140	1122	118	.56	4.93	(0.51)
28 to 29	819	54	48	135	1056	102	.59	4.17	(0.46)
29 to 30	722	82	35	154	993	117	.61	5.22	(0.58)
...									
(0 to 12	126	168	167	78	539	335	.88	[655.52	(141.0)
12 to 30	228	147	157	89	621	304	.82)	43.53	(8.19)]

Individual Actors' Prominence

(Centralities)





		DEGREES										INDEGREES										OUTDEGREES									
		Dichotomous					Valued					Dichotomous					Valued					Dichotomous					Valued				
	m	total	avg.	s.d.	cent	s.d.	total	avg.	s.d.	cent	s.d.	total	avg.	s.d.	cent	s.d.	total	avg.	s.d.	cent	s.d.	total	avg.	s.d.	cent	s.d.	total	avg.	s.d.	cent	s.d.
Us	22	830	37.7	10.8	.081	.016	3839	174.5	49.0	.171	.059	419	19.1	5.1	.082	.019	2048	93.1	27.5	.184	.069	411	18.7	5.8	.079	.015	1791	81.4	22.6	.159	.050
Ge	30	1060	35.3	9.1	.074	.009	3926	130.9	20.7	.135	.025	517	17.2	4.9	.071	.008	1943	64.8	12.0	.133	.026	543	18.1	4.4	.076	.011	1983	66.1	9.6	.137	.026
Sw	30	909	30.3	9.7	.067	.013	2014	67.1	9.5	.069	.010	455	15.2	4.9	.066	.014	1020	34.0	3.8	.070	.010	454	15.1	5.1	.067	.013	994	33.1	6.3	.068	.012
Fi	30	806	26.9	10.5	.059	.011	1787	59.6	18.8	.060	.017	405	13.5	5.4	.058	.011	863	28.8	8.2	.058	.013	401	13.4	5.5	.059	.016	924	30.8	11.1	.062	.021
Dk	30	701	23.4	11.9	.051	.008	1235	41.2	16.6	.041	.012	359	12.0	5.8	.051	.007	635	21.2	8.1	.042	.013	342	11.4	6.3	.050	.011	600	20.0	9.2	.039	.013
Uk	12	268	22.3	8.5	.046	.024	453	37.8	21.4	.047	.032	137	11.4	4.5	.047	.026	247	20.6	12.4	.052	.036	131	10.9	4.1	.045	.023	206	17.2	9.4	.042	.028
Be	30	627	20.9	9.0	.048	.007	820	27.3	4.3	.028	.005	315	10.5	4.7	.047	.007	402	13.4	2.6	.028	.005	312	10.4	4.5	.049	.010	418	13.9	2.3	.029	.006
It	28	567	20.3	8.3	.044	.023	1382	49.4	32.2	.049	.032	313	11.2	4.8	.047	.021	717	25.6	15.6	.050	.030	254	9.1	4.0	.041	.026	665	23.8	17.1	.047	.035
Br	19	362	19.1	7.0	.033	.007	568	29.9	12.8	.030	.011	175	9.2	3.8	.032	.009	275	14.5	7.1	.029	.013	187	9.8	3.7	.034	.009	293	15.4	6.2	.031	.010
Tw	14	261	18.6	6.0	.033	.008	320	22.9	7.6	.024	.007	104	7.4	4.4	.025	.009	121	8.6	5.2	.017	.007	157	11.2	2.6	.042	.014	199	14.2	3.8	.031	.011
Se	27	500	18.5	10.3	.046	.019	952	35.3	20.3	.037	.019	245	9.1	5.4	.044	.019	457	16.9	10.1	.035	.019	255	9.4	5.2	.048	.020	495	18.3	10.4	.038	.020
Co	24	443	18.5	6.4	.035	.007	888	37.0	8.4	.037	.007	214	8.9	3.2	.033	.009	420	17.5	4.7	.035	.007	229	9.5	3.3	.036	.007	468	19.5	4.0	.039	.008
Hn	24	443	18.5	4.4	.035	.007	856	35.7	6.4	.036	.008	218	9.1	2.7	.035	.007	413	17.2	4.1	.035	.009	225	9.4	1.9	.036	.008	443	18.5	3.1	.038	.009
Cr	28	514	18.4	8.4	.037	.008	1039	37.1	14.6	.038	.015	272	9.7	4.1	.039	.009	532	19.0	7.2	.039	.015	242	8.6	4.5	.036	.009	507	18.1	7.7	.037	.016
Jp	30	542	18.1	7.6	.042	.016	855	28.5	10.1	.029	.010	259	8.6	4.8	.041	.021	415	13.8	6.5	.028	.013	283	9.4	3.3	.044	.013	440	14.7	4.1	.030	.009
Is	30	535	17.8	7.5	.041	.011	854	28.5	8.2	.029	.008	267	8.9	3.6	.040	.009	412	13.7	3.6	.028	.007	268	8.9	4.2	.041	.015	442	14.7	5.5	.030	.011
Bo	30	517	17.2	7.0	.038	.009	1072	35.7	8.3	.038	.013	263	8.8	3.7	.039	.011	529	17.6	4.4	.038	.013	254	8.5	3.5	.036	.009	543	18.1	4.3	.039	.014
Fr	26	447	17.2	9.9	.032	.016	726	27.9	15.2	.029	.018	230	8.9	4.9	.033	.016	361	13.9	7.6	.029	.018	217	8.4	5.1	.031	.016	365	14.0	8.0	.029	.019
Mx	30	502	16.7	7.3	.038	.011	1007	33.6	11.8	.034	.011	247	8.2	4.2	.040	.015	491	16.4	6.5	.033	.011	255	8.5	3.3	.037	.010	516	17.2	5.8	.035	.011
Kr	20	326	16.3	6.6	.028	.008	443	22.2	10.0	.021	.007	149	7.5	3.7	.025	.009	197	9.9	5.4	.019	.008	177	8.9	3.1	.031	.008	246	12.3	4.9	.024	.007
Sp	10	151	15.1	8.6	.022	.010	192	19.2	10.8	.017	.008	87	8.7	4.1	.026	.010	107	10.7	5.5	.020	.008	64	6.4	4.7	.018	.012	85	8.5	5.6	.015	.009
Au	28	419	15.0	8.8	.030	.009	536	19.1	7.6	.019	.007	213	7.6	4.4	.031	.010	273	9.8	3.9	.020	.007	206	7.4	4.6	.028	.011	263	9.4	4.2	.019	.008
Sl	11	163	14.8	6.4	.023	.009	268	24.4	11.9	.021	.010	74	6.7	3.8	.021	.011	126	11.5	6.6	.020	.011	89	8.1	3.1	.025	.008	142	12.9	5.7	.022	.010
In	25	355	14.2	6.7	.027	.013	649	26.0	14.0	.027	.017	153	6.1	4.2	.023	.014	277	11.1	7.9	.023	.017	202	8.1	3.4	.030	.015	372	14.9	7.5	.032	.020
Ke	17	224	13.2	5.5	.024	.010	298	17.5	7.1	.019	.009	114	6.7	2.9	.025	.011	156	9.2	3.9	.020	.010	110	6.5	2.9	.024	.010	142	8.4	3.8	.018	.008
Gh	26	341	13.1	6.6	.026	.008	578	22.2	9.1	.022	.008	158	6.1	3.3	.024	.008	278	10.7	5.1	.021	.010	183	7.0	3.7	.028	.010	300	11.5	4.7	.023	.008
Sa	1	13	13.0	–	.027	–	16	16.0	–	.015	–	6	6.0	–	.025	–	8	8.0	–	.015	–	7	7.0	–	.029	–	8	8.0	–	.015	–
Po	6	77	12.8	5.2	.018	.010	80	13.3	5.7	.011	.006	40	6.7	2.5	.019	.010	42	7.0	3.0	.012	.006	37	6.2	2.8	.017	.010	38	6.3	2.9	.011	.006
Li	8	99	12.4	4.1	.023	.007	141	17.6	7.8	.016	.006	46	5.8	2.6	.021	.009	68	8.5	4.4	.015	.007	53	6.6	2.0	.024	.007	73	9.1	3.8	.016	.006
Nl	5	61	12.2	8.6	.020	.018	92	18.4	20.7	.017	.020	30	6.0	4.3	.020	.018	45	9.0	10.4	.016	.020	31	6.2	4.3	.020	.018	47	9.4	10.3	.017	.020
Nz	30	364	12.1	6.4	.030	.011	496	16.5	5.3	.017	.005	231	7.7	3.3	.037	.014	322	10.7	3.6	.023	.011	133	4.4	4.1	.022	.016	174	5.8	4.1	.011	.008
Gu	1	12	12.0	–	.025	–	16	16.0	–	.015	–	6	6.0	–	.025	–	9	9.0	–	.017	–	6	6.0	–	.025	–	7	7.0	–	.013	–
Hk	1	11	11.0	–	.022	–	13	13.0	–	.012	–	6	6.0	–	.025	–	7	7.0	–	.013	–	5	5.0	–	.020	–	6	6.0	–	.011	–
No	20	218	10.9	6.8	.018	.009	242	12.1	8.0	.011	.007	109	5.5	3.6	.018	.010	112	5.6	3.7	.010	.006	109	5.5	3.4	.018	.009	130	6.5	4.7	.012	.008
Ng	23	246	10.7	5.1	.019	.006	341	14.8	7.7	.014	.006	100	4.4	2.7	.015	.007	140	6.1	3.9	.011	.006	146	6.4	2.8	.023	.008	201	8.7	4.4	.017	.008
Ni	1	10	10.0	–	.020	–	11	11.0	–	.010	–	5	5.0	–	.020	–	6	6.0	–	.011	–	5	5.0	–	.020	–	5	5.0	–	.009	–
Ca	8	74	9.3	13.1	.013	.019	104	13.0	19.5	.011	.016	45	5.6	8.1	.016	.024	61	7.6	11.6	.013	.020	29	3.6	5.0	.010	.014	43	5.4	7.9	.009	.013
Ur	1	8	8.0	–	.011	–	8	8.0	–	.007	–	4	4.0	–	.011	–	4	4.0	–	.007	–	4	4.0	–	.011	–	4	4.0	–	.007	–
At	18	137	7.6	6.6	.031	.023	266	14.8	10.6	.015	.011	80	4.4	3.8	.032	.024	146	8.1	6.0	.017	.013	57	3.2	3.1	.029	.023	120	6.7	5.0	.014	.011
Ph	6	38	6.3	8.6	.035	.020	105	17.5	12.8	.019	.013	18	3.0	3.8	.033	.027	53	8.8	7.1	.019	.015	20	3.3	4.9	.037	.021	52	8.7	6.1	.019	.011
Nc	3	12	4.0	4.6	.009	.011	15	5.0	6.2	.005	.007	10	3.3	3.5	.016	.017	13	4.3	5.1	.009	.011	2	0.7	1.2	.003	.006	2	0.7	1.2	.002	.003
Id	5	19	3.8	6.4	.023	.022	44	8.8	8.5	.011	.010	8	1.6	3.1	.023	.022	20	4.0	3.9	.009	.009	11	2.2	3.4	.024	.024	24	4.8	4.8	.012	.012
Tu	2	6	3.0	4.2	.006	.009	7	3.5	5.0	.003	.005	3	1.5	2.1	.006	.009	3	1.5	2.1	.003	.004	3	1.5	2.1	.006	.009	4	2.0	2.8	.004	.005
Ru	4	11	2.8	5.5	.006	.012	13	3.3	6.5	.004	.008	7	1.8	3.5	.007	.015	9	2.3	4.5	.006	.011	4	1.0	2.0	.004	.008	4	1.0	2.0	.003	.005
Np	4	10	2.5	5.0	.005	.011	10	2.5	5.0	.003	.006	2	0.5	1.0	.002	.004	2	0.5	1.0	.001	.002	8	2.0	4.0	.009	.017	8	2.0	4.0	.005	.010
Ug	4	8	2.0	4.0	.004	.008	12	3.0	6.0	.004	.007	3	0.8	1.5	.003	.006	6	1.5	3.0	.004	.007	5	1.3	2.5	.005	.011	6	1.5	3.0	.004	.007
Mz	4	4	1.0	2.0	.002	.004	6	1.5	3.0	.002	.004	3	0.8	1.5	.003	.006	5	1.3	2.5	.003	.006	1	0.3	0.5	.001	.002	1	0.3	0.5	.001	.001
Ma	4	3	0.8	1.5	.002	.003	3	0.8	1.5	.001	.002	2	0.5	1.0	.002	.004	2	0.5	1.0	.001	.002	1	0.3	0.5	.001	.002	1	0.3	0.5	.001	.001
Lt	4	2	0.5	1.0	.001	.002	2	0.5	1.0	.001	.001	1	0.3	0.5	.001	.002	1	0.3	0.5	.001	.001	1	0.3	0.5	.001	.002	1	0.3	0.5	.001	.001
Th	4	2	0.5	1.0	.001	.002	2	0.5	1.0	.001	.001	2	0.5	1.0	.002	.004	2	0.5	1.0	.001	.002	0	0.0	0.0	.000	.000	0	0.0	0.0	.000	.000
Pt	2	0	0.0	0.0	.000	.000	0	0.0	0.0	.000	.000	0	0.0	0.0	.000	.000	0	0.0	0.0	.000	.000	0	0.0	0.0	.000	.000	0	0.0	0.0	.000	.000

The Dyad Census

1. $M = \sum_{i>j} x_{ij}\, x_{ji}$
2. $A = \sum_{i>j} \left[(1 - x_{ij})\, x_{ji} + x_{ij}\, (1 - x_{ji}) \right]$
3. $N = \sum_{i>j} (1 - x_{ij})\, (1 - x_{ji})$
4. $M + A + N = \binom{n}{2}$

	Mutual	Asymmetrical	Null	Total
t_0	89	67	169	325
t_1	NA	NA	NA	NA
t_2	38	29	38	105
t_3	52	26	42	120
t_4	56	36	61	153
t_5	87	24	60	171
t_6	71	36	64	171
t_7	105	20	85	210
t_8	79	49	125	253
t_9	91	53	181	325
t_{10}	126	12	187	325
t_{11}	100	39	161	300
t_{12}	101	44	233	378
t_{13}	113	41	197	351
t_{14}	134	39	205	378
t_{15}	175	8	195	378
t_{16}	111	51	216	378
t_{17}	131	73	231	435
t_{18}	138	67	201	406
t_{19}	158	33	305	496
t_{20}	177	37	251	465
t_{21}	150	74	241	465
t_{22}	162	34	300	496
t_{23}	164	33	154	351
t_{24}	85	51	215	351
t_{25}	83	73	169	325
t_{26}	84	65	202	351
t_{27}	66	83	412	561
t_{28}	63	57	475	595
t_{29}	60	69	399	528
t_{30}	75	86	367	528

1. $\hat{\gamma}_1 = \frac{2M}{A}$

2. $\hat{\gamma}_2 = \frac{A}{2N}$

Simple stochastic model

(Wasserman, 1980)

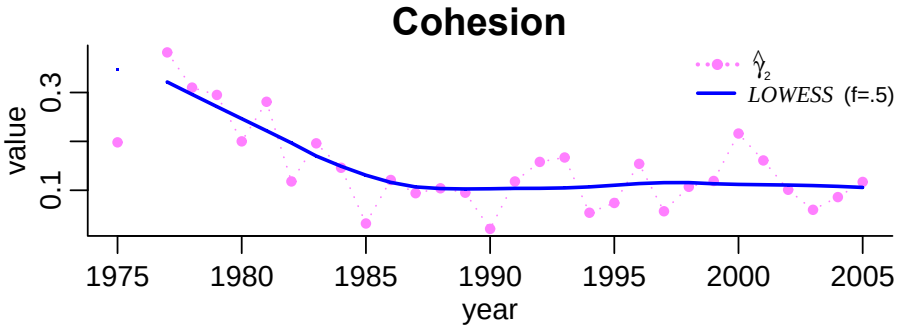
$$P\{Y_{ij} = y_1, Y_{ji} = y_2\} = \frac{\exp[y_1(\mu + \alpha_i + \beta_j) + y_2(\mu + \alpha_j + \beta_i) + y_1 y_2 \rho]}{k_{ij}}$$

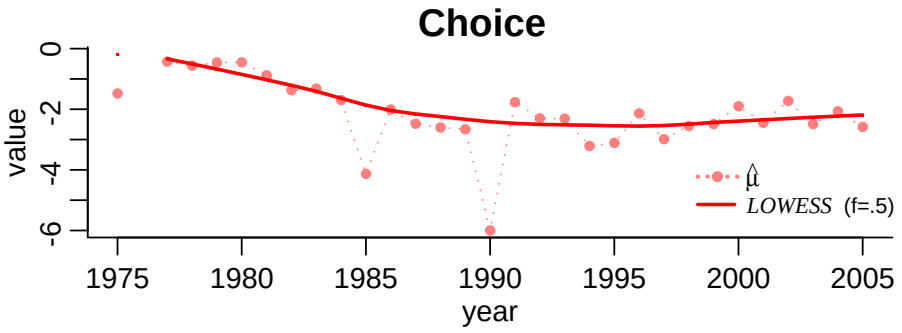
$$k_{ij} = 1 + \exp(\mu + \alpha_i + \beta_j) + \exp(\mu + \alpha_j + \beta_i) + \exp(2\mu + \alpha_i + \beta_j + \alpha_j + \beta_i + \rho)$$

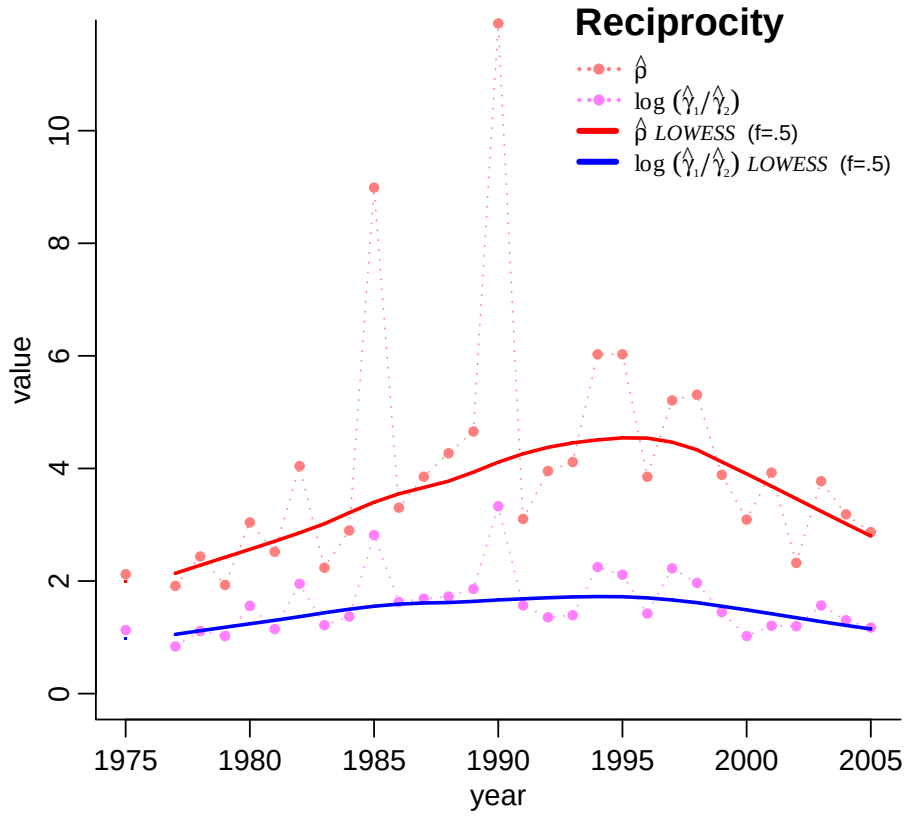
The p_1- model

(Holland & Leinhardt, 1981)

Network Effects







Geographical distribution of ties

	EU	LA	AP	AF	NA
EU	17	15	9	6	4
LA	15	1	1	0	2
AP	9	1	3	0	2
AF	7	0	0	0	1
NA	4	2	1	1	0

	EU	LA	AP	AF	NA
EU		0	0	-1	0
LA	0		0	0	0
AP	0	0		0	+1
AF	+1	0	0		0
NA	0	0	-1	0	

①	EU					LA					AP					AF					NA				
time	EU	LA	AP	AF	NA	EU	LA	AP	AF	NA	EU	LA	AP	AF	NA	EU	LA	AP	AF	NA	EU	LA	AP	AF	NA
t00	54	26	25	2	10	26	5	1	1	5	21	0	31	3	7	2	1	2	0	1	10	5	6	1	0
	.22	.11	.10	.01	.04	.11	.02	.00	.00	.02	.09	.00	.13	.01	.03	.01	.00	.01	.00	.00	.04	.02	.02	.00	.00
t01	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
t02	26	10	10	-	7	11	1	2	-	1	13	0	9	-	3	-	-	-	-	-	7	2	3	-	0
	.25	.10	.10		.07	.10	.01	.02		.01	.12	.00	.09		.03						.07	.02	.03		.00
t03	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
t30	41	44	30	25	-	39	0	5	2	-	22	2	6	0	-	17	1	2	0	-	-	-	-	-	-
	.17	.19	.13	.11		.17	.00	.02	.01		.09	.01	.03	.00		.07	.00	.01	.00						
total	1347	1181	763	490	215	1210	45	104	29	115	734	96	210	27	82	415	22	22	7	53	220	125	91	54	0
avg.	44.9	39.4	25.4	18.1	9.8	40.3	1.5	3.5	1.1	5.2	24.5	3.2	7.0	1.0	3.7	15.4	0.8	0.8	0.3	2.8	10.0	5.7	4.1	2.8	0.0
	.17	.15	.09	.07	.04	.15	.01	.01	.00	.02	.09	.01	.03	.00	.01	.06	.00	.00	.00	.01	.04	.02	.02	.01	.00
s.d.	13.2	15.5	6.6	8.5	3.0	14.3	1.6	2.1	1.1	3.3	6.5	2.2	5.2	1.0	1.5	7.5	1.0	0.7	0.8	2.0	3.3	3.2	1.7	1.7	0.0
	.05	.04	.02	.03	.02	.04	.01	.01	.00	.01	.02	.01	.03	.01	.01	.02	.00	.00	.00	.01	.02	.01	.01	.01	.00

EU	E	F	G	H
E	102	112	21	296
F	119	9	1	138
G	13	1	0	10
H	321	108	12	84

	E	F	G	H	total
E	19	21	4	56	100
F	45	3	0	52	100
G	54	4	0	42	100
H	61	21	2	16	100

①	E				F				G				H			
time	E	F	G	H	E	F	G	H	E	F	G	H	E	F	G	H
t00	13	3	-	13	5	0	-	2	-	-	-	-	11	3	-	4
	.24	.06		.24	.09	.00		.04					.20	.06		.07
t01	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
t02	5	-	-	7	-	-	-	-	-	-	-	-	7	-	-	7
	.19	.00		.27	.00	.00		.00					.27	.00		.27
t03	1	3	-	9	1	0	-	2	-	-	-	-	10	3	-	4
	.03	.09		.27	.03	.00		.06					.30	.09		.12
t04	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
t30	2	2	2	7	4	0	0	1	4	1	0	3	8	2	3	2
	.05	.05	.05	.17	.10	.00	.00	.02	.10	.02	.00	.07	.20	.05	.07	.05

EU = { E: (Au, Be, Ge, Nl, Ni, Sw, Uk), F: (Fr, It, Pt, Sp), G: (Lt, Po, Ru, Tu), H: (Dk, Fi, Is, No, Se) }

$$f_i(x(i \rightsquigarrow j)) + g_i(x, j) + U(j)$$

$$f_i(\beta, x) = \sum_{K=1}^{L} \beta_k \, s_{ik}(x)$$

1. $s_{i1}(x) = \sum_j x_{ij}$
2. $s_{i2}(x) = \sum_j x_{ij} \, x_{ji}$
3. $s_{i3}(x) = \sum_j x_{ij} \sqrt{\sum_h x_{hj}}$
4. $s_{i4}(x) = \sum_j x_{ij} \sqrt{\sum_h x_{jh}}$
5. $s_{i5}(x) = \sum_j x_{ij} \, (1 - |u_i - u_j|)$

Evaluation function

$$g_i(\gamma, x, j) = \sum_{h=1}^{H} \gamma_h \, r_{ijh}(x)$$

1. $r_{ij1}(x) = x_{ij} \, x_{ji}$
2. $r_{ij2}(x) = x_{ij} \, (1 - |u_i - u_j|)$

Endowment function

Stochastic Actor-oriented model

(Snijders, 2001 and 2005)

Effect	model 1a			model 2a			model 3a		
	par.	(s.e.)	*p*-value	par.	(s.e.)	*p*-value	par.	(s.e.)	*p*-value
Outdegree	-2.24	(.10)	<.001	-2.36	(.09)	<.001	-2.53	(.08)	<.001
Reciprocity	1.90	(.05)	<.001	1.93	(.05)	<.001	2.65	(.16)	<.001
*Popularity of alter**	.29	(.10)	<.005	.39	(.02)	<.001	.44	(.02)	<.001
Activity of alter*	.06	(.12)	<.50						
COVARIATES									
Regions similarity				-0.35	(.10)	<.001	-0.91	(.27)	<.001
Regions sim. × recip.				.03	(.18)	<.30			
ENDOWMENT									
Reciprocity							-1.41	(.29)	<.001
Regions similarity							1.17	(.54)	<.03

* Squared.

(a) All Regions with geographical characteristics ($n = 51$).

Effect	model 1b			model 2b			model 3b		
	par.	(s.e.)	*p*-value	par.	(s.e.)	*p*-value	par.	(s.e.)	*p*-value
Outdegree	-1.995	(.22)	<.001	-1.22	(.09)	<.001	-1.22	(.09)	<.001
Reciprocity	1.83	(.15)	<.001	1.93	(.14)	<.001	1.89	(.30)	<.001
Popularity of alter*	.21	(.23)	<.40						
Activity of alter*	.22	(.23)	<.40						
COVARIATES									
Cultural similarity				-0.48	(.17)	<.01	-0.69	(.26)	<.01
Cultural sim. × recip.				-0.06	(.28)	<.50			
ENDOWMENT									
Reciprocity							.11	(.54)	<.50
Cultural similarity							.37	(.45)	<.50

* Squared.

(b) European Region with cultural characteristics ($n = 20$).

Effect	model 1a			model 2a			model 3a			model 3a †		
	par.	(s.e.)	*p*-value	par.	(s.e.)	*p*-value	par.	(s.e.)	*p*-value	par.	(s.e.)	*p*-value
Outdegree	-2.24	(.10)	<.001	-2.36	(.09)	<.001	-2.53	(.08)	<.001	-2.34	(.09)	<.001
Reciprocity	1.90	(.05)	<.001	1.93	(.05)	<.001	2.65	(.16)	<.001	2.60	(.37)	<.001
*Popularity of alter**	.29	(.10)	<.005	.39	(.02)	<.001	.44	(.02)	<.001	.34	(.03)	<.001
Activity of alter*	.06	(.12)	<.50									
COVARIATES												
Regions similarity				-0.35	(.10)	<.001	-0.91	(.27)	<.001	-0.96	(.26)	<.001
Regions sim. × recip.				.03	(.18)	<.30						
ENDOWMENT												
Reciprocity							-1.41	(.29)	<.001	-1.40	(.73)	<.03
Regions similarity							1.17	(.54)	<.03	1.23	(.52)	<.011
Transitivity										.01	(.00)	<.001

* Squared.

(a) All Regions with geographical characteristics ($n = 51$).

Effect	model 1b			model 2b			model 3b			model 3b †		
	par.	(s.e.)	*p*-value	par.	(s.e.)	*p*-value	par.	(s.e.)	*p*-value	par.	(s.e.)	*p*-value
Outdegree	-1.995	(.22)	<.001	-1.22	(.09)	<.001	-1.22	(.09)	<.001	-1.62	(.10)	<.001
Reciprocity	1.83	(.15)	<.001	1.93	(.14)	<.001	1.89	(.30)	<.001	1.71	(.13)	<.001
Popularity of alter*	.21	(.23)	<.40									
Activity of alter*	.22	(.23)	<.40									
COVARIATES												
Cultural similarity				-0.48	(.17)	<.01	-0.69	(.26)	<.01	.09	(.00)	<.001
Cultural sim. × recip.				-0.06	(.28)	<.50						
ENDOWMENT												
Reciprocity							.11	(.54)	<.50			
Cultural similarity							.37	(.45)	<.50			
Transitivity										-0.84	(.12)	<.001

* Squared.

(b) European Region with cultural characteristics ($n = 20$).

- What are the main results?

  1) Positional systems suggested a centre-periphery pattern
     - by uncovering actors' roles and positions in the network …

  2) Statistical model provided evidence for heterophily
     - by testing the homophily principle in the formation of ties …

- Points:
  - Positional systems change can be assessed through the Rand index
  - General and particular indices used to see trajectories over time
  - Structural zeroes & missing values to record data when order changes

- Acknowledgement

Pajek v.1.16; StOCNET v.1.8: SIENA v.3.1, p_2 v.1.3; R v.2.7.2: blockmodeling v.0.1.2, sna v.1.2, nettheory v.2.0, network v.1.0-1, stats v.2.3.1, graphics v.2.3.1, nlme v.3.1-73.